

# Useful Formulae for Spectroscopy

## CALCULATION OF MAXIMUM ALLOWABLE PRESSURES ON CIRCULAR WINDOW MATERIALS

Calculation of maximum pressure will depend upon a number of user selectable parameters. For instance, the window material, window size, flange size and a safety factor all may be varied depending upon application.

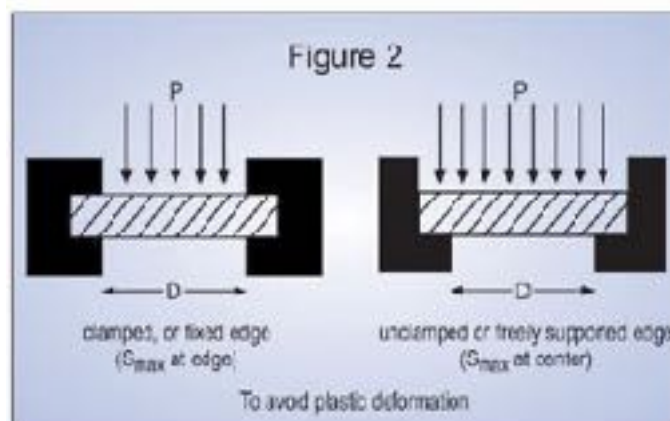
To calculate maximum allowable pressure, we must assume that the maximum stress in a uniform circular plate is given by the equation:

$$S_{max} = (k \times D^2 \times P) / (4 \times t^2)$$

where k is a constant, the value for which depends upon whether or not the window is clamped — use 0.75 for clamped windows and 1.125 for unclamped (See Fig. 2).  $S_{max}$  is the maximum stress, D is the window diameter under pressure (ie, the portion of window not supported by the flange as shown in the schematic in Figure 2), P is the load (expressed in psi), and t is the thickness of the window material. In the formula solving for t, the window diameter D can be expressed in any unit of measure such as mm or inches. To avoid plastic deformation a safety factor must be introduced where SF is a safety factor and  $F_a$  is apparent elastic limit for the material itself. Where apparent elastic limit is not available, use yield stress. Allowing for a safety factor (SF), the equations for calculation of maximum allowable pressure and for minimum window thickness (t) where the operating pressure (P) is known are:

$$P = 4 (F_a \times t^2) / (SF \times k \times D^2)$$

$$t = \sqrt{\frac{SF \times P \times K \times D^2}{4 \times F_a}}$$



The apparent elastic limits of some IR optical materials are listed below:

CaF <sub>2</sub>	5300	KCl	330
BaF <sub>2</sub>	3900	KBr	160
KRS-5	3800	CsI	810
NaCl	350	MgF <sub>2</sub>	7200

## ATR SPECTROSCOPY

The formulae set forth below are useful for ATR (MIR) spectroscopy.

### Penetration Depth

The depth of penetration is defined by the formula:

$$dp = \frac{\lambda}{2\pi n_1 (\sin^2 \Phi - n_{21}^2)^{1/2}}$$

where  $\lambda$  is the wavelength of the infrared light,  $n_1$  is the refractive index of the ATR crystal,  $\Phi$  is the angle of incidence of the infrared beam at the boundary and  $n_{21}$  is the ratio of the refractive indices of the sample,  $n_s$ , and ATR crystal,

$$n_c \text{ and } n_{21} = \frac{n_s}{n_c}$$

Since the evanescent wave decreases in intensity exponentially from the surface of the crystal, the penetration depth,  $dp$ , is defined as the distance at which the amplitude of this wave has decreased to  $(1/e)$  or 37% of its original value.



## Effective Angle of Incidence

This is the angle of incidence of the infrared beam internally in the ATR crystal when a variable angle HATR such as the Varimax™ is used for analysis. When the scale angle,  $\Phi_{\text{scale}}$ , is not equal to the crystal face angle,  $\Phi_{\text{face}}$ , the effective angle,  $\Phi$  is different than the scale angle due to refraction.

$$\Phi_{\text{actual}} = \Phi_{\text{scale}} - \sin^{-1} \left[ \frac{\sin (\Phi_{\text{scale}} - \Phi_{\text{face}})}{n_{\text{crystal}}} \right]$$

$n_{\text{crystal}}$  = refractive index

## Number of Reflections

The number of reflections in the crystal gives a measure of the intensity of the resulting spectrum. This number is a function of the effective angle of incidence  $\Phi$ , and the length,  $l$ , and thickness,  $t$ , of the crystal.

$$N = \frac{l}{t \cot \Phi}$$

## Effective Pathlength

The effective pathlength,  $P_{\text{eff}}$ , is defined as the product of the penetration depth  $d_p$ , and the number of bounces,  $N$ , the IR beam makes within the crystal:

$$P_{\text{eff}} = d_p \times N$$

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